

PE.A-89-3-5

-----1	-1
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-200 $\rightarrow \sigma_x$	-200
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250 $\rightarrow \sigma_y$	250
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175 $\rightarrow \tau_{xy}$	175
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$\frac{\sigma_x + \sigma_y}{2}$	25
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$\frac{\sigma_x - \sigma_y}{2}$	-225
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-----2	-2
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$\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$	310.044
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$\frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$	-260.044
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$$\frac{1}{2} \cdot \frac{\tan^{-1}\left(\frac{\frac{\sigma x \sigma y}{2}}{\frac{\sigma x - \sigma y}{2}}\right) \cdot 180}{\pi}$$

-18.9375

-----3

-3

$$\sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \sigma x \sigma y^2}$$

285.044

$$\frac{1}{2} \cdot \tan^{-1}\left(\frac{\frac{-(\sigma x - \sigma y)}{2}}{\sigma x \sigma y}\right) \cdot \frac{180}{\pi}$$

26.0625

-----*tensor*

-tensor

$$\begin{bmatrix} -200 & 175 \\ 175 & 250 \end{bmatrix} \rightarrow \sigma$$

$$\begin{bmatrix} -200 & 175 \\ 175 & 250 \end{bmatrix}$$

$$\text{eigvl}(\sigma) \rightarrow ev$$

{-260.044,310.044}

$$\frac{ev[2] - ev[1]}{2}$$

285.044

$$\begin{bmatrix} ev[2] & 0 \\ 0 & ev[1] \end{bmatrix} \rightarrow \sigma_{12}$$

$$\begin{bmatrix} 310.044 & 0 \\ 0 & -260.044 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \rightarrow q$$

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\sigma_{12} = q \cdot \sigma \cdot q^T \rightarrow eq$$

$$\begin{bmatrix} 310.044 = 350 \cdot \sin(\theta) \cdot \cos(\theta) + 450 \cdot (\sin(\theta))^2 - 200 & 0 = 350 \cdot (\cos(\theta))^2 + 450 \cdot \sin(\theta) \cdot \cos(\theta) - 175 \\ 0 = 350 \cdot (\cos(\theta))^2 + 450 \cdot \sin(\theta) \cdot \cos(\theta) - 175 & -260.044 = 450 \cdot (\cos(\theta))^2 - 350 \cdot \sin(\theta) \cdot \cos(\theta) - 200 \end{bmatrix}$$

$$\text{solve}(eq[2,1], \theta)$$

$$\theta = -8.1845 \text{ or } \theta = -6.61371 \text{ or } \theta = -5.04291 \text{ or } \theta = -3.47211 \text{ or } \theta = -1.90132 \text{ or } \theta = -0.330522 \text{ or } \theta = 1.24027 \text{ or } \theta = 2.81107$$

$$\frac{-0.33052158 \cdot 180}{\pi}$$

$$-18.9375$$

$$-18.937491571996 + 45$$

$$26.0625$$

[]

$$\text{-----} -1$$

$$-1$$

$$\begin{bmatrix} 200 & 80 \\ 80 & -100 \end{bmatrix}$$

$$\begin{bmatrix} 200 & 80 \\ 80 & -100 \end{bmatrix}$$

$$\text{-----} -2$$

$$-2$$

$$\triangle \text{ solve } \left(\det \left(\begin{bmatrix} 200-\sigma & 80 \\ 80 & -100-\sigma \end{bmatrix} \right) = 0, \sigma \right)$$

$$\sigma = -120 \text{ or } \sigma = 220$$

$$\text{eigVl} \left(\begin{bmatrix} 200 & 80 \\ 80 & -100 \end{bmatrix} \right)$$

$$\{ 220., -120. \}$$

$$\frac{200-100}{2}$$

$$50$$

$$\frac{200+100}{2}$$

$$150$$

$$\sqrt{\left(\frac{200+100}{2} \right)^2 + 80^2}$$

$$170$$

$$\frac{200-100}{2} + \sqrt{\left(\frac{200+100}{2} \right)^2 + 80^2}$$


$$220$$

$$\frac{200-100}{2}-\sqrt{\left(\frac{200+100}{2}\right)^2+80^2}$$

-120

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14-applied-1

 solve $\left(\det \left(\begin{bmatrix} x-50 & 0 & -40 \\ 0 & x-40 & 0 \\ -40 & 0 & x-10 \end{bmatrix} \right) = 0, x \right)$

$$x = -70 \text{ or } x = -40 \text{ or } x = 30$$

$$\frac{1}{2} \cdot \frac{\tan^{-1} \left(\frac{2 \cdot -40}{-50-10} \right) \cdot 180}{\pi}$$

$$26.565051$$

[]

-----1

-1

$$\frac{-\sqrt{2} \cdot 90 \cdot 10^3}{100^2} \rightarrow \sigma_x$$

$$-9 \cdot \sqrt{2}$$

$$\frac{-\sqrt{2} \cdot 90 \cdot 10^3}{100^2} \rightarrow \sigma_y$$

$$-9 \cdot \sqrt{2}$$

$$0 \rightarrow \sigma_z$$

$$0$$

-----2

-2

$$\frac{\sigma_x - \nu \cdot (\sigma_y + \sigma_z)}{e} \big|_{e=20000 \text{ and } \sigma_z=0 \text{ and } \nu=0.1} \rightarrow \varepsilon_x$$

$$-0.000573$$

$$\frac{\sigma_y - \nu \cdot (\sigma_x + \sigma_z)}{e} \big|_{e=20000 \text{ and } \sigma_z=0 \text{ and } \nu=0.1} \rightarrow \varepsilon_y$$

$$-0.000573$$

$$\frac{\sigma_z - \nu \cdot (\sigma_x + \sigma_y)}{e} \big|_{e=20000 \text{ and } \sigma_z=0 \text{ and } \nu=0.1} \rightarrow \varepsilon_z$$

$$0.000127$$

-----3

-3

$$\varepsilon_x + \varepsilon_y + \varepsilon_z \rightarrow \varepsilon_v$$

$$-0.001018$$

-----4

-4

$$\frac{\sigma x \cdot \varepsilon x + \sigma y \cdot \varepsilon y + \sigma z \cdot \varepsilon z}{2}$$

0.00729

-----5

-5

$$100^3 \cdot \frac{\sigma x \cdot \varepsilon x + \sigma y \cdot \varepsilon y + \sigma z \cdot \varepsilon z}{2}$$

7290.

[]

-----1

1

$$\frac{\varepsilon x + \varepsilon y}{2} + \frac{\varepsilon x - \varepsilon y}{2} \cdot \cos(2 \cdot \theta) + \frac{\gamma_{xy}}{2} \cdot \sin(2 \cdot \theta) \rightarrow \varepsilon$$

$$\frac{\gamma_{xy} \cdot \sin(2 \cdot \theta)}{2} + \varepsilon x \cdot \left(\frac{\cos(2 \cdot \theta)}{2} + \frac{1}{2} \right) + \varepsilon y \cdot \left(\frac{1}{2} - \frac{\cos(2 \cdot \theta)}{2} \right)$$

$$\text{solve} \left(\begin{cases} \varepsilon = \varepsilon a | \theta = 0 \\ \varepsilon = \varepsilon b | \theta = 60^\circ \\ \varepsilon = \varepsilon c | \theta = 120^\circ \end{cases}, \{ \varepsilon x, \varepsilon y, \gamma_{xy} \} \right)$$

$$\gamma_{xy} = \frac{2 \cdot (\varepsilon b - \varepsilon c) \cdot \sqrt{3}}{3} \text{ and } \varepsilon x = \varepsilon a \text{ and } \varepsilon y = \frac{-(\varepsilon a - 2 \cdot (\varepsilon b + \varepsilon c))}{3}$$

$$\gamma_{xy} = \frac{2 \cdot (\varepsilon b - \varepsilon c) \cdot \sqrt{3}}{3} \text{ and } \varepsilon x = \varepsilon a \text{ and } \varepsilon y = \frac{-(\varepsilon a - 2 \cdot (\varepsilon b + \varepsilon c))}{3} | \varepsilon a = 0.001 \text{ and } \varepsilon b = -0.001 \text{ and } \varepsilon c = 0.001$$

$$\gamma_{xy} = -0.002309 \text{ and } \varepsilon x = 0.001 \text{ and } \varepsilon y = -0.000333$$

-----2

-2

$$\frac{\varepsilon x + \varepsilon y}{2} + \sqrt{\left(\frac{\varepsilon x - \varepsilon y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2} | \gamma_{xy} = -0.0023094010767585 \text{ and } \varepsilon x = 0.001 \text{ and } \varepsilon y = -3.33333333333333\text{E-}4$$

0.001667

$$\frac{\varepsilon x + \varepsilon y}{2} - \sqrt{\left(\frac{\varepsilon x - \varepsilon y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2} | \gamma_{xy} = -0.0023094010767585 \text{ and } \varepsilon x = 0.001 \text{ and } \varepsilon y = -3.33333333333333\text{E-}4$$

-0.001

$$\frac{\frac{1}{2} \cdot \tan^{-1} \left(\frac{\frac{\gamma_{xy}}{2}}{\frac{\epsilon x - \epsilon y}{2}} \right) \cdot 180}{\pi} \quad \gamma_{xy} = -0.0023094010767585 \text{ and } \epsilon x = 0.001 \text{ and } \epsilon y = -3.33333333333333\text{E-}4 \quad -30.$$

$$\text{-----} -3 \quad -3$$


$$\text{solve} \left(\left\{ 0.001666666666666666 \cdot e = \sigma 1 - \nu \cdot \sigma 2, \{ \sigma 1, \sigma 2 \} \right\} \middle| e = 30000 \text{ and } \nu = 0.3 \right) \quad \sigma 1 = 45.0549 \text{ and } \sigma 2 = -16.4835$$

$$\text{-----} -4 \quad -4$$

$$\text{eigVl} \left(\begin{bmatrix} 0.001 & \frac{-0.002309}{2} \\ \frac{-0.002309}{2} & -3.33\text{E-}4 \end{bmatrix} \right) \quad \{ 0.001667, -0.001 \}$$

$$\begin{bmatrix} 0.001667 & 0 \\ 0 & -0.001 \end{bmatrix} = \begin{bmatrix} \cos(2 \cdot \theta) & \sin(2 \cdot \theta) \\ -\sin(2 \cdot \theta) & \cos(2 \cdot \theta) \end{bmatrix} \cdot \begin{bmatrix} 0.001 & \frac{-0.002309}{2} \\ \frac{-0.002309}{2} & -3.33\text{E-}4 \end{bmatrix} \cdot \begin{bmatrix} \cos(2 \cdot \theta) & \sin(2 \cdot \theta) \\ -\sin(2 \cdot \theta) & \cos(2 \cdot \theta) \end{bmatrix}^T \rightarrow eq1$$

$$\begin{cases} 0.001667 = 0.001333 \cdot (\cos(2 \cdot \theta))^2 - 0.002309 \cdot \sin(2 \cdot \theta) \cdot \cos(2 \cdot \theta) - 0.000333 & 0 = -0.002309 \cdot (\cos(2 \cdot \theta))^2 - 0.001333 \cdot \sin(2 \cdot \theta) \cdot \cos(2 \cdot \theta) \\ 0 = -0.002309 \cdot (\cos(2 \cdot \theta))^2 - 0.001333 \cdot \sin(2 \cdot \theta) \cdot \cos(2 \cdot \theta) + 0.001155 & -0.001 = 0.002309 \cdot \sin(2 \cdot \theta) \cdot \cos(2 \cdot \theta) + 0.0 \end{cases}$$

 solve(eq1[2,1],θ)| $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

$$\theta = -0.261808 \text{ or } \theta = 0.523591$$

$$\frac{0.523591 \cdot 180}{\pi}$$

$$29.9996$$



10-applied-4

-----1

1

$$\frac{100 \cdot 300^3}{12} \rightarrow i$$

2.25E8

$$\frac{200000}{2 \cdot (1+0.3)} \rightarrow g$$

76923.1

$$\frac{80}{3} \cdot 1 \cdot 10^6 \rightarrow mc$$

2.66667E7

$$100^3 \rightarrow q$$

1000000

$$\frac{80}{3} \cdot 10^3 \rightarrow vc$$

26666.7

-----2

-2

$$\frac{mc \cdot 50}{i} \rightarrow \sigma x$$

5.92593

$$0 \rightarrow \sigma y$$

0

$$\frac{vc \cdot q}{i \cdot 100} \rightarrow \alpha \gamma \quad 1.18519$$

$$\text{-----} -3 \quad 3$$

$$\frac{\alpha \gamma}{g} \rightarrow \gamma \quad 0.000015$$

$$\text{solve}\left(\left\{\begin{array}{l} e \cdot \varepsilon x = \sigma x - 0.3 \cdot 0 \\ e \cdot \varepsilon y = \sigma y - 0.3 \cdot \sigma x \end{array}\right\}, \{\varepsilon x, \varepsilon y\}\right) | e = 200000 \quad \varepsilon x = 0.00003 \text{ and } \varepsilon y = -0.000009$$

$$2.9629629629631\text{E-}5 \rightarrow \varepsilon x \quad 0.00003$$

$$-8.8888888888893\text{E-}6 \rightarrow \varepsilon y \quad -0.000009$$

$$\frac{\varepsilon x + \varepsilon y}{2} + \frac{\varepsilon x - \varepsilon y}{2} \cdot \cos(2 \cdot 45^\circ) + \frac{\gamma}{2} \cdot \sin(2 \cdot 45^\circ) \quad 0.000018$$

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PE.C-117-2-3

$$\text{-----} -1 \qquad -1$$

$$300 \cdot 400 \rightarrow a \qquad 120000$$

$$\frac{400 \cdot 300^3}{12} \rightarrow ix \qquad 900000000$$

$$\frac{300 \cdot 400^3}{12} \rightarrow iy \qquad 1600000000$$

$$300 \cdot 10^3 \cdot -50 \rightarrow mx \qquad -15000000$$

$$300 \cdot 10^3 \cdot 90 \rightarrow my \qquad 27000000$$

$$\text{-----} -2 \qquad -2$$

$$\frac{300 \cdot 10^3}{a} + \frac{mx}{ix} \cdot y + \frac{my}{iy} \cdot x \rightarrow q \qquad \frac{27 \cdot x}{1600} - \frac{y}{60} + \frac{5}{2}$$

$$\text{-----} -3 \qquad -3$$

$$\text{solve}(q=0, x) | y=0 \qquad x=-148.148$$

$$\text{solve}(q=0, y) | x=0 \qquad y=150$$

-----4

-4

$q|x=-200$ and $y=150$

-3.375

$q|x=200$ and $y=-150$

8.375

[]

-----1

1

-----2

2

$$\begin{bmatrix} 15 & 23 & 34 \\ 23 & -30 & 18 \\ 34 & 18 & -25 \end{bmatrix} \cdot 10^{-6} \rightarrow e$$

$$\begin{bmatrix} 0.000015 & 0.000023 & 0.000034 \\ 0.000023 & -0.00003 & 0.000018 \\ 0.000034 & 0.000018 & -0.000025 \end{bmatrix}$$

$$\text{⚠ -expand}(\det(e - x \cdot \text{identity}(3)))$$

$$1 \cdot x^3 + 0.00004 \cdot x^2 - 0.000000002084 \cdot x - 8.2447\text{E-}14$$

$$-4 \cdot 10^{-5} \rightarrow i1$$

$$\frac{-1}{25000}$$

$$-2.084 \cdot 10^{-9} \rightarrow i2$$

$$-0.000000002084$$

$$8.24467 \cdot 10^{-14} \rightarrow i3$$

$$8.24467\text{E-}14$$

-----3

3

$$\frac{3 \cdot i2 - i1^2}{9} \rightarrow q$$

$$-872.444444444\text{E-}12$$

$$\frac{2 \cdot i1^3 - 9 \cdot i1 \cdot i2 + 27 \cdot i3}{54} \rightarrow r$$

$$24.9596462963\text{E-}15$$

$$\cos^{-1}\left(\frac{r}{\sqrt{-q}^3}\right) \rightarrow \theta \quad 251.373683666\text{E-}3$$

$$2 \cdot \sqrt{-q} \cdot \cos\left(\frac{\theta}{3}\right) + \frac{i1}{3} \rightarrow \varepsilon 1 \quad 4.55337494992\text{E-}5$$

$$2 \cdot \sqrt{-q} \cdot \cos\left(\frac{\theta+2 \cdot \pi}{3}\right) + \frac{i1}{3} \rightarrow \varepsilon 2 \quad -4.70486094942\text{E-}5$$

$$2 \cdot \sqrt{-q} \cdot \cos\left(\frac{\theta+4 \cdot \pi}{3}\right) + \frac{i1}{3} \rightarrow \varepsilon 3 \quad -3.8485140005\text{E-}5$$

$$\text{-----}4 \quad 4$$

$$\text{expand}\left(\det\left(\begin{bmatrix} 15 \cdot \mu - x & 23 \cdot \mu & 34 \cdot \mu \\ 23 \cdot \mu & -30 \cdot \mu - x & 18 \cdot \mu \\ 34 \cdot \mu & 18 \cdot \mu & -25 \cdot \mu - x \end{bmatrix}\right)\right) \quad -x^3 - 40 \cdot \mu \cdot x^2 + 2084 \cdot \mu^2 \cdot x + 82447 \cdot \mu^3$$

$$\text{solve}\left(-x^3 - 40 \cdot \mu \cdot x^2 + 2084 \cdot \mu^2 \cdot x + 82447 \cdot \mu^3 = 0, x\right) | \mu = 10^{-6} \\ x = -4.70482310828\text{E-}5 \text{ or } x = -3.84855569834\text{E-}5 \text{ or } x = 4.55337880662\text{E-}5$$

□